

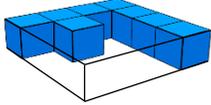
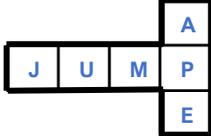
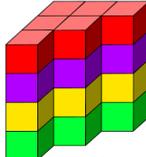
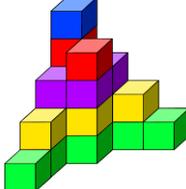
# Math Challenge #13

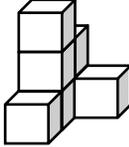


First Name: _____	Last Name: _____	Grade: _____
Teacher: _____	Parent's email: _____	

## Cubes and Cuboids

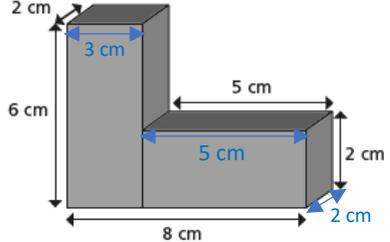
**Kinder & First Grade: solve at least 3 problems.**  
**Second & Third Grade: solve at least 7 problems.**  
**Fourth Grade and above: solve at least 12 problems.**

	<i>Answer</i>
1. Bottom layer has 4 cubes and top layer has 2 cubes. Total cube is $4 + 2 = 6$ .	6
2. The total cubes to make the tower is 9. Since Damien has 5 cubes already, he then needs $9 - 5 =$ <b>4 more cubes.</b>	4
3. $3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 24$ or $3 \times 8 = 24$	24
4. The layer is measured 4 cubes by 4 cubes, so the total would be $4 \times 4 = 16$ cubes. Since there are 8 cubes already, we need additional <b>8 cubes</b> to complete the layer.	8
	
5. $8 \times 3 + 4 = 28$	28
6. The best way to solve these kind of questions is to cut out a real life version of the cube pattern and fold it yourself. Once you fold the cut-out, you will see that the opposite to the letter P is letter U.	U
	
7.  A large cube measuring 4 by 4 by 4 cubes has a total of $4 \times 4 \times 4 = 64$ cubes. There are 10 cubes already, so we need $64 - 10 = 54$ cubes.	<i>54 [cubes]</i>
8. Each layer in Fig. A has 6 cubes. After exchanging 2 red cubes for 2 green cubes, and gaining 1 blue cube, he has: 8 green cubes, 6 yellow cubes, 6 purple cubes, 4 red cubes, and 1 blue cube. In Fig. B, there are 8 green cubes, 6 yellow cubes, 4 purple cubes, 2 red cubes, and 1 blue cube. He did not use <b>2 red cubes and 2 purple cubes.</b>	<i>2 red cubes and 2 purple cubes</i>
 	
9. Looking down from the top, we will see 3 faces, from the bottom, also 3 faces. From the front we see 3 faces, from the left we see 3 faces, from the right 3 faces, and from the back also 3 faces. Total square faces: $3+3+3+3+3+3 = 18$	<i>18 [square faces]</i>
	

10.  Looking down from the top, we see 4 faces, and from the bottom also 4 faces. From the front we see 4 faces, from the left side we see 6 faces, from the right side 6 faces, and from the back 4 faces. Total square faces with paint on them:  $4+4+4+6+6+4 = 28$

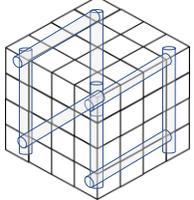
*28 [square faces]*

11. Volume =  $(3 \times 6 \times 2) + (5 \times 2 \times 2) = 36 + 20 = 56 \text{ cm}^3$



*56 cm<sup>3</sup>*

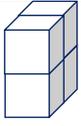
12. By drilling a hole in each corner of the face that is facing him, he will have 16 cubes that have been drilled through.



When he drills through from the top all the way through the bottom, 2 cubes on each edge have already been drilled so they don't count. A total of 8 extra cubes have been drilled. So,  $8 + 16 = 24$  cubes are drilled and  $64 - 24 = 40$  cubes are not drilled.

*40*

13. Each cube has 6 identical faces, which have area  $24 \div 6 = 4 \text{ cm}^2$ . The cuboid has 16 of these same square faces, and thus has area  $16 \times 4 \text{ cm}^2 = 64 \text{ cm}^2$



*64 cm<sup>2</sup>*

14. The brick is a rectangular prism, so it has 12 edges. We list all different integers for its edges and the sum has to be 36 inches. So, possible measurements for the brick are:

Width (4 edges)	Length (4 edges)	Height (4 edges)	Volume
1	2	6	<b>12 in<sup>3</sup></b>
1	3	5	<b>15 in<sup>3</sup></b>
2	3	4	<b>24 in<sup>3</sup></b>

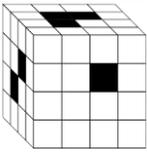
Notice that when you interchange the width, length, and height of one measurement, it produces the same volume.

Another way to solve:  
 The sum of all edges is 36 inches, therefore,  $4 \times (l + w + h) = 36$ . Thus,  $l + w + h = 9$ .  
 To get the sum of 9 with 3 different integers, we can have these measurements which results in 3 possible volumes:  
 1 by 2 by 6 ( $V = 12 \text{ in}^3$ ), 1 by 3 by 5 ( $V = 15 \text{ in}^3$ ), and 2 by 3 by 4 ( $V = 24 \text{ in}^3$ ).

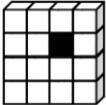
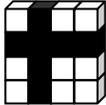
*3*

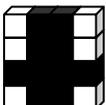
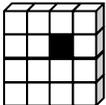
15. The volume of the Fig. A (yellow cuboid is  $6 \times 8 \times 20 = 960 \text{ cm}^3$   
 The height of the Fig. B (red cuboid) is  $960 \div (10 \times 8) = 12 \text{ cm}$

*12 [cm]*

16.  Before any of the cubes were removed, there were  $4 \times 4 \times 4 = 64$  cubes. Cutting from top to bottom, we remove  $3 \times 4 = 12$  smaller cubes. Cutting from front to back, we remove  $4 - 1$  (overlapped with the previously removed cube) = 3 smaller cubes, cutting from left to right, we remove  $2 \times 4 - 4 = 4$  cubes. So, there are  $12 + 3 + 4 = 19$  cubes removed.

Another way to count the cubes by layers (cross-sections).

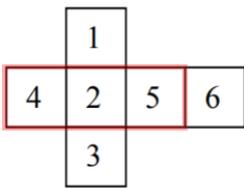
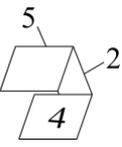
From the front layer, 1 cube removed, 15 cubes remained.  Next layer: 7 removed, 9 cubes remained 

Next layer: 10 removed, 6 cubes remained  Last layer: 1 removed, 15 cubes remained 

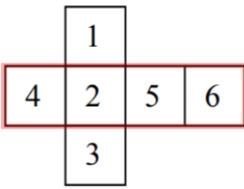
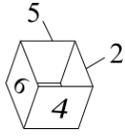
In total  $1 + 7 + 10 + 1 = 19$  removed, or  $64 - (15 + 9 + 6 + 15) = 19$  cubes were removed.

17. The sum of the numbers on all the faces is 21.  $1+2+3+4+5+6 = 21$ . The first observation implies that the sum of the numbers on a couple of opposite sides is 8. This couple can be (6; 2) or (5; 3). The second observation implies that the sum of the numbers on another couple of opposite sides is 9. That couple can be (6; 3) or (5; 4). Combining the above information, the only possibility is that (6; 2) and (5; 4) are on opposite sides. Hence, (3; 1) is the third couple of opposite sides, therefore the answer is 3.

18. We could list all possible products and find the largest possible one from the list. But we could also start working on possible large products. To achieve the largest possible products, we can start by considering the larger numbers. The 4, 5 and 6 could result in the largest possible products, so we can begin by seeing whether they touch.

  The 4 and the 5 do not touch, as shown in the diagram below, which shows part of the folded cube. Therefore, we can eliminate 4 and 5 (they don't touch, and they don't meet at a vertex).

The 6 touches the 4 and the 5, as shown.

  This means that the greatest product must include the 6 and the 5 or the 6 and the 4.

If we look at the 6 and the 5, then include the 3, not the 1, the product will be  $6 \times 5 \times 3 = 90$ .

Solution is available on April 22, 2022, at [www.mathinaction.org](http://www.mathinaction.org)