Math Challenge #4

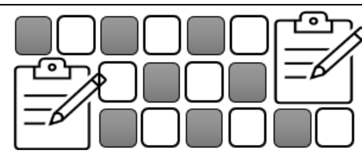
First Name:	____________________	**Last Name:**	______________________________	**Grade:**	________
Teacher:	______________________	**Parent's email:**	____		

Problems with Cards

Kinder & First Grade: solve <u>at least</u> 3 problems.
Second & Third Grade: solve <u>at least</u> 7 problems.
Fourth Grade and above: solve <u>at least</u> 12 problems.

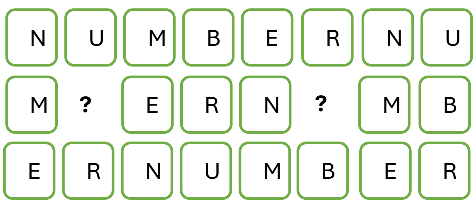
Answer

1. Ron has gray and white cards. He arranges the cards in a specific pattern as below. Two notepads are placed on top of some of these cards. How many gray cards are hidden beneath these two notepads? 2+2 = **4**



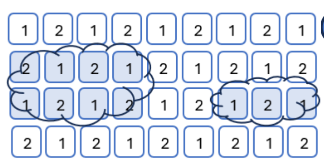
Answer: *4*

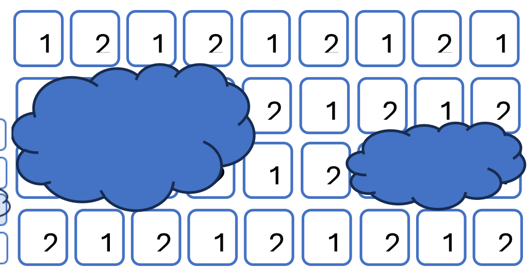
2. A number of cards (with letters N, U, M, B, E, R) are arranged to continuousely spelled the word 'NUMBER'.
Which letter cards are missing?
The cards are letters on the word 'NUMBER'. The missing cards are **B** and **U**.



Answer: *B and U*

3. Lisa has a stack of cards with either the number 1 or 2 on each card. She arranges the cards in a specific pattern as shown. What is the sum of the numbers covered by the clouds?
The cards that are covered by the large cloud: 2+1+2+1+1+2+1+2 = 12. The cards that are covered by the small cloud: 1+2+1 = 4. Total sum: 12 + 4 = **16**.



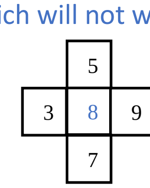


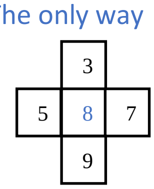
Answer: *16*

4. James has a set of cards that are numbered and stacked in order from 1 to 9. James and his sister take turns taking one card from the stack until there are no more cards left. If James starts first (taking the number 1 card), what is the sum of the numbers on his sister's cards?
James will have card 1, 3, 5, 7, and 9. His sister will have card 2, 4, 6, 8. Total on her cards is 2+4+6+8 = **20**

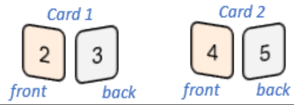
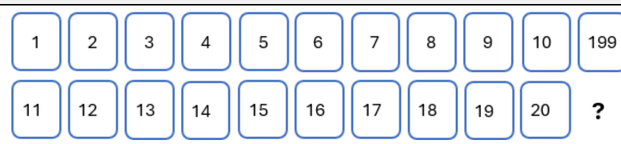
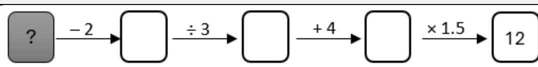
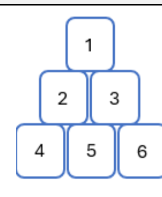
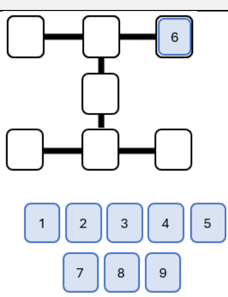
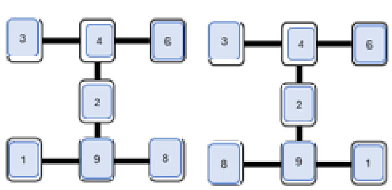
Answer: *20*

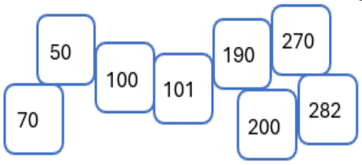
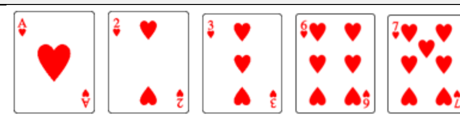
5. The sum of all the numbers on the cards she has is 3 + 5 + 7 + 8 + 9 = 32. The card that will go to the center will be used twice for both column and the row. At this moment we counted the number that goes in the center only once. So let's add a number and see whether the result is divisible by 2 (one for column, one for row).
32 + 3 = 35, odd, so no, 3 is not in the center; 32 + 5 = 37, odd, so no, 5 is not in the center.
If you add any odd number to 32, you will get an odd result, which will not work. The only way to do it is by placing the card with 8 in the middle. 32 + 8 = 40, even. So, the row and the column, each has a sum of 40/2 = 20.
In the row with the sum 20, 8 is in the middle, thus the other two numbers must add up to 12, it is 3 and 9 or 5 and 7.





Answer: *8*

6.	 Possibilities: 24, 25, 34, 35, 42, 43, 52, 53. There are **8 unique two digit numbers**.	*8 [ways]*
7.	 There are 7 numbers that are covered: 14+24+25+26+35+36+46 = **206**	*20 6*
8.	Jordyn cards: 1, _ _, 4, 5, 6, _ _ _ __, 11, 12, 13, 14, 15, __ __ __ __ __ → sum: 81 Her brother, Tim: _, 2, 3, _ _ _, 7, 8, 9, 10, __ __ __ __ __, 16, 17, 18, 19, 20 → sum: 129 The difference of Tim's sum and Jordyn's sum: 129 – 81 = **48**	*Tim, 48*
9.	82 – 28 = 54. The sum of the numbers of the two cards: 5 + 4 = **9**	*9*
10	Total cards: (3 boxes × 50) + (5 boxes × 40) + 50 = 150 + 200 + 50 = **400** cards	*400 [baseball cards]*
11.	One way: comparing the first 10 cards on each row, their sum differs by 100. The sum of the numbers in each row is the same. Therefore, the missing card is 199 - 100 = **99**.  Another way: the sum of the first row: 55+199 = 254. The total of each row is the same. The missing card: 254 – (31×5) = 254 – 155 = **99.**	*99*
12.	 Work backward: 12 ÷ 1.5 = 8, 8 – 4 = 4, 4×3 = 12, 12 + 2 = **14** Check: 14 – 2 = 12, 12 ÷ 3 = 4, 4+4 = 8, 8 × 1.5 = 12 ✓	*14*
13.	The fourth row: 7, 8, 9, 10 *(4 cards)* The fifth row: 11, 12, 13, 14, 15 *(5 cards)* The sixth row: 16, 17, 18, …, 21 *(6 cards)* The seventh row: 22, 23, …, 28 The eight row: 29, 30 , …, 36 The ninth row: 37, 38, …, 45 The tenth row: 46, 47, …, 55 *(10 cards)* The sum: 46+47+48+49+50+51+52+53+54+55 = 101×5 = **505** *Notice that there's a pattern.* • *The first number in each row: 1, 2, 4, 7, 11, 16, …* • *The first row has 1 number, second row has 2 numbers, third row has 3 numbers, and so on.*  	*505*
14.	Since she cannot remove the blue card, we can calculate the one blue card to be 25%. To have 75% red cards, she needs only 3 cards. So, Pam must remove 59 – 3 = **56** red cards.	*56 [red cards]*
15.	 *Possible placements:*  Out of the 8 proposed cards we can't use 5 or 7, because no matter where we place them they will be used in the product of only one or at most two numbers. And we must have all three products the same. Now let's look on the prime factorization of the numbers that are left: 1, 2, 3, 2^2, 2^3, 3^2. We have in the right corner 6 = 2×3. This means that each line should have at least one 3. But we have 3 and 9. In the first line left corner we can place 3. Which means the top row will have a product of two threes in it. So, the vertical line and horizontal line must also have 9 in them. The only place that work is the center of the last row. Now we need to balance 2, 4, and 8. 2×4 = 1×8, the 6 has a factor of 2 inside, so to get the 8 in the first row we will place 4 in the center of it. Then to get 8 in the vertical row we need to place 2 in the middle. The last row can have either 1 on the left, 8 on the right or vice versa. Each line will have a product of **72**.	*72*

16.	 There are 4 aces in a deck of 52 cards. P (First Ace) = 4/52 = 1/13. After drawing the first ace, there are now 3 aces left in a deck of 51 cards (no longer 52). P (Second Ace, First Ace) = 3/51 = 1/17. So, the probability that both cards are aces: P (Two Aces) = 1/13 * 1/17 = **1/221**.	*1/221*
17.	 In order to have the sum of the cards taken half of the leftover cards, the sum has to be a third of the sum of all cards. The sum of all the cards is 70 + 50 + 100 + 101 + 190 + 270 + 200 + 282 = 1263. 1/3 of 1263 = 421. To get the largest number of cards with this sum, Diana needs to take smaller numbers. There is only 1 card that has 1 in units place, so she must use 101. 421 – 101 = 320. So, it could be 101 + 50 + 270, but these are only 3 cards. Since she took **4 cards** with a sum of 421, the cards are: 50+70+101+200, and the greatest number on the cards is **200**.	*200*
18.	If there's no restrictions, we will have a total of 5! or 120 arrangements.  We then can calculate out the invalid arrangements: When Ace is in first position, the rest of the cards can be shuffled in: 4! = 24 ways. When 7 is in the last position, the rest of the cards can be shuffled in: 4! = 24 ways. When both Ace is in the first position and 7 is in the last position, the three cards in the middles can be shuffled in: 3! = 6 ways. The number of ways where the Ace is never in the first position and the 7 is never in the last position: 120 – (24+24-6) = **78** ways.	*78 [ways]*

Solution is available on November 8, 2024
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