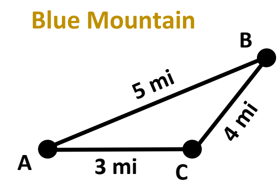
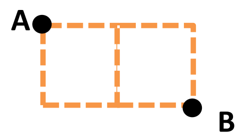
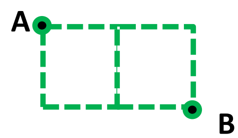
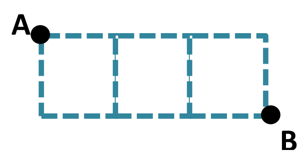
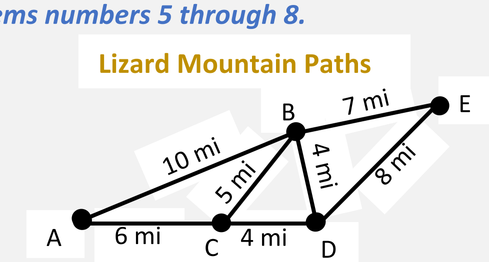
Math Challenge #9

Paths

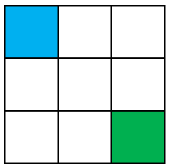
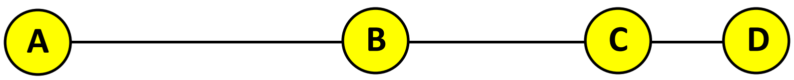
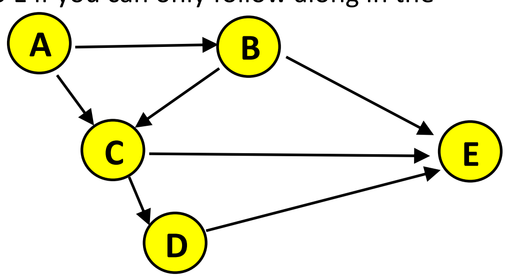
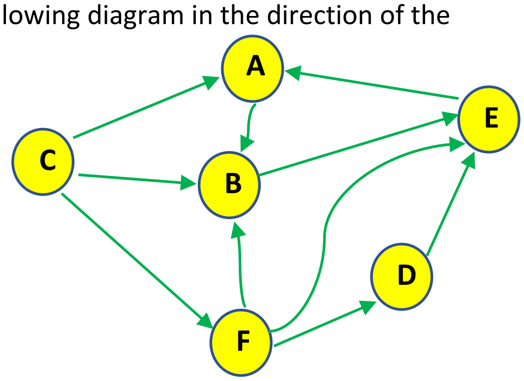
Kinder & First Grade: solve at least 3 problems.
Second & Third Grade: solve at least 7 problems.
Fourth Grade and above: solve at least 12 problems.

		Answer
1.	Kelly went on a hike at the Blue Mountain. She went from point A to C then to B. She came back down directly from point B to A. What was the total distance of her hike? 3 miles + 4 miles + 5 miles = 12 miles 	*12 [miles]*
2.	Anishka would like to get to point B from point A. How many ways are there to get from A to B if she can only stay on the dotted lines and not walk over the same line more than once? 	*4 [ways]*
3.	The paths from A to B are made by joining 2 squares. If the side of each square measures 6 yards long, how many yards is the shortest path from A to B following the dotted lines? 	*18 [yards]*
4.	Three squares are joined together. The side of each square measures 9 feet long. a. What is the shortest path, in feet, you can take from A to B if you follow the dotted lines? 36 feet b. What is the longest path, in feet, you can take from A to B if you follow the dotted lines and you do not walk over the same line more than once? 54 feet 	*a. 36 [feet]* *b. 54 [feet]*

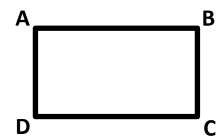
Use this picture to solve problems numbers 5 through 8.



5.	Tom plans to bike from point A to point E. What is the shortest distance he can take?	*17 [miles]*
6.	What is the longest distance from point A to point E if you are visiting a point not more than once?	*27 [miles]*
7.	How many ways are there to get from point A to point E, if you can only visit a point once? (Hint: list all possibilities in an organized way) ABE, ABCDE, ABDE, ACBE, ACDE, ACBDE, ACDBE	*7 [ways]*

8.	Maya, on her bike, is leaving Point A and wants to visit points B, C, D, and E, but not necessarily in that order. What is the shortest distance that Maya can take if she has to stay on the paths? ACDBE : 6 + 4 + 4 + 7 = 21 miles	*21 [miles]*
9.	Supposed that you want to get from the blue to green square. Each step we may move right or down only. How many distinct paths can you take?  Right-right-down-down Down-down-right-right Right-down-right-down Down-right-down-right Right-down-down-right Down-right-right-down	*6 [paths]*
10.	 Town A to town D is 46 miles. Town B to town C is 16 miles. If the distance from town C to town D is half the distance from Town B to C, how many miles is it traveling from Town A to C? The distance from town B to C is half of 16 miles = 8 miles. The distance from town A to C is 46 – 8 = 38 miles	*38 [miles]*
11.	How many distinct paths are there from A to E if you can only follow along in the direction of the arrows? Note that you may not visit a point more than once.  ABE ACE ABCE ABCDE ACDE Therefore, there are only 5 distinct paths.	*5 [paths]*
12.	If you must follow along the paths of the following diagram in the direction of the arrows, how many paths are there from C to E? Note that you may not visit a point more than once.  From point *C*, we can take the path either to point *A*, *B*, or *F*. The path leading from *C* to *A* only has one further option. We can't go directly to *E*, as we can only travel past *A* to *B*. Likewise, of the four paths connecting to *B*, only one can leave from *B*. This path goes to our destination, *E*. This means that we currently have **one successful path**, and this is the only path that begins with *C* going to *A*. Next, let's look at the path beginning from *C* to *B*. Once again, only one path can leave from *B*, and we found in the previous paragraph that only one path leaves *B*. Thankfully, it goes to our destination, *E*, giving us **two successful paths**, and this is the only path that begins with *C* going to *B*. Last, let's look at the path beginning from *C* to *F*. *F* has three paths leading away from it: one to *B*, one to *E*, and one to *D*. The one to *E* takes us straight to our destination, giving us **three successful paths**. The one to *B* behaves the same as in the past two paragraphs, giving us **four successful paths**. The only remaining path leads to *D*, and there is only one path leading from *D*, which is again going to our destination, *E*, giving us **five successful paths**. **Therefore, there are five paths leading from *C* to *E*.**	*5 [paths]*

13. Ayush walks along the edges of a rectangular pool from point A to B to C to D, a distance of 38 meters. Aaron walks along the edges of the same pool from B to C to D to A, a distance of 31 meters. What is the perimeter of the pool, in meters?

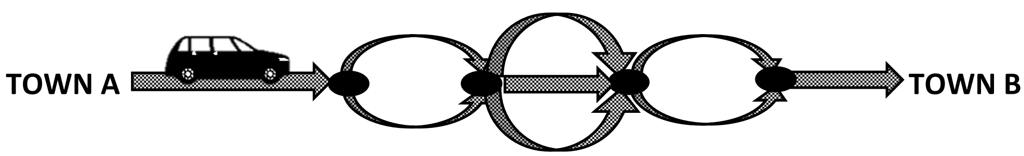


Together they cover a total of 3 lengths and 3 widths; since 38 +31 = 69 meters, we have:

- 3 lengths + 3 widths = 69
- 1 length + 1 width = 23
- 2 lengths + 2 widths (which is the perimeter) = 46 meters.

46 [meters]

14. To go from Town A to Town B, a car can take different paths illustrated below:

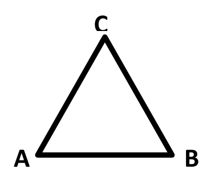


How many different paths can the car go from Town A to Town B if it can only go on one-way roads (indicated by arrows), and it cannot pass any intersection (dot) more than once?

There are 2 roads to choose from the first dot to the second, 3 roads from the second dot to the third, and 2 roads from the third to the road leading to Town B: 2x3x2 = 12 paths.

12 [paths]

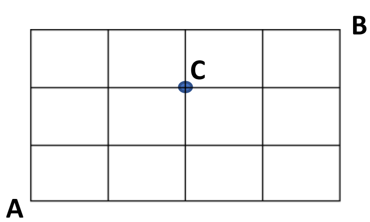
15. Sidhart travels from city A to city B to city C and back to city A. Each city is 120 miles from the other two. His average rate from city A to city B is 60 mph. His average rate from city B to city C is 40 mph. His average rate from city C to city A is 24 mph. What is Sidhart's average rate for the entire trip, in miles per hour?



Divide the total distance traveled by the total time needed: 360 ÷ (2+3+5) = 36 mph.

36 [mph]

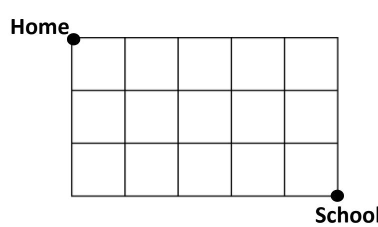
16. How many different paths are there from A to B that go through the point C in the following lattice path? Note that at each point you can **only move up or to the right**.

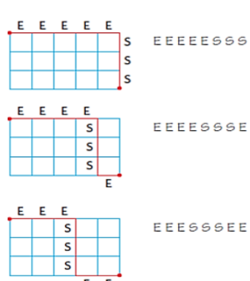


Every path from A to B going through the point C can be broken into two: a path from A to C and a path from C to B. To go from A to C, we need 2 moves going up and 2 moves going right. The path is determined by the order we arrange these four moves. The number of different paths on this is 6. Similarly, we have 3 paths from C to B. So, by the multiplication principle, the number of paths from A to B (going through point C) is 6 × 3 = 18.

18 [paths]

17. Caitlin's home is three blocks north and five blocks west of her school. How many routes can Caitlin take from home to school if she always travels either south or east?





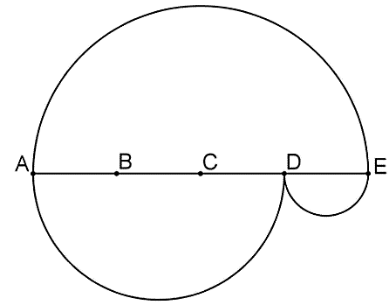
Notice that in each of the possible routes shown, the total number of blocks traveled is 8. Also notice that some elements that are identical to each other, E shows up 5 times and S shows up 3 times.

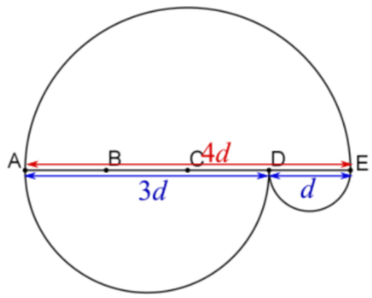
Thus, we must treat this as a permutation problem in which some of the elements are identical.

$P = 8!/(5! \times 3!) = 56$

56 [routes]

18. In the diagram, AE is divided into four equal parts. Semicircles have been drawn with AE, AD and DE as diameters. This has created two new paths from A to E, an upper path and a lower path.
Which path is shorter, the upper or the lower? How do you know?





Let the diameter of the smallest semicircle be *d*. Then *d* is equal to one of the equal parts of AE, and the diameters of the other semicircles are 3*d* and 4*d*.
The upper path is half of the circumference of a full circle with diameter 4*d*. The circumference would $\pi \times 4d = 4\pi d$, so the length of the path is $2\pi d$.
The lower path is the distance around a semicircle with diameter 3*d* and the distance around a semicircle with diameter *d*.
The distance around the medium semicircle is $\frac{1}{2}\pi \times 3d = 3/2\pi d$.
The distance around the smallest semicircle is $\frac{1}{2}\pi \times d = \frac{1}{2}\pi d$.
So the total length of the lower path is $3/2\pi d + 1/2\pi d = 4/2\pi d = 2\pi d$.
So the two paths are of equal length.

None.
The two paths are of equal length.

Solution is available on February 19, 2021 at www.mathinaction.org